

Chapter 9

The Special Theory of Relativity

9.1 Pre-History of concepts about light

It is interesting to note that so much of our understanding of the physical universe is based on our interpretations of the operation of vision and light and how dramatically this has changed over the centuries. The very earliest descriptions were usually attempts to understand the process of vision. As is so often the case, these early attempts to produce a theory of vision goes back to ancient Greece and is based on a simple idea of the extension of our sense of touch. We feel the location and texture of surfaces by contact. The corresponding idea of Empedocles and Euclid was that vision involved the emanation from the eye of rays that sensed the surface and returned to the eye, much like fingers. This simple picture is still with us in the form of the special vision of comic book heros like Superman and in expressions such as “stop staring at me,” which implies the something is coming from the eye. It was the philosophical school based on atomism that lead up to Aristotle that first clearly established the vision is based on the emanations from the seen object basically by noting that there was no vision in the dark. It was also now possible to join the ideas of vision with the more general issue of light. The greek development reached a pinnacle in the ability of Ptolemy to describe reflection and measure refraction. []

After the fall of the greek nation states and during the dark ages in the west, arabian scholars not only rescued the greek texts but they continued the development of the ray theory of light. Alkindi and Alhazen bringing together the greek ideas and extending them to lenses and mirrors and Alhazen

producing what was to become the classic text on optics, *Kitah al-manazir* or *The Book of Optics*. An excellent review of the ancient contributions to optics and a layman's review of current ideas is given in the book by David Parks, [Park 1997].

With the renaissance, the primary issue became the nature of the emissions and, following Galileo, a much more refined effort to carefully measure the properties of light. Descartes filled all space with a particulate essence that was the basis for subsequent particle theories of light including Newton's. A wealth of experiments by Boyle, Hooke, and Young revealed the important properties of interference and diffraction and lead to the ideas of a particulate basis being displaced by the wave theory that originated with Huygens but reached its complete expression with Fresnel. In hind sight, it is interesting that phenomena associated with the polarization of light was the major difficulty in the acceptance of the wave theory. The development of the wave theory is very well articulated in [Buchwald 1989].

In the classical, pre-quantum, period, the next great contribution to our understanding of optical phenomena came as an addendum to Maxwell's effort to unify the electric and magnetic force systems. His development of a field theory of fundamental forces and the identification of light as the long range traveling solutions of his dynamical equations for the electromagnetic forces provided a new foundation for understanding all the phenomena associated with light. It was the anomalies associated with this dynamic and the requirement of Galilean relativity that Poincaré, Lorentz, and Einstein used to discover the basis for the special theory of relativity. A short history of these developments is given in [Born & Wolf 1999]. In Volume II of his history of the development of theories of the the electric force, Whittaker provides a detailed and somewhat unique perspective of the development of special relativity, [Whittaker 1953] A more conventional history is given by Pais, [Pais 1982].

Although we are not concerned here except incidentally with the modern theory of light as expressed by quantum field theory, any complete account of our understanding of light must include the work associated with Planck and Einstein and later developed by Feynman, Schwinger, and Tomonaga, [Feynman 1985]. A detailed development of these ideas are in the later chapters of these notes.

9.2 Galilean Invariance

Almost anyone who has sat quietly waiting to depart from a bus depot or a dock and has had the bus or boat gently start to leave has had the experience of feeling that it is the depot or dock that has moved away. This simple physiological phenomena has its basis in a very general physical law that was first articulated by Galileo and thus is called Galilean Invariance. It is one of the most striking and far reaching of all of the laws of physics. It is impossible to over emphasize its importance; it is the basis of our understanding of space-time and motion. The simplest statement of the law is that there is no experiment that can be performed that can measure a uniform velocity. Since we can only know what can be measured, we can never know how fast we are moving. There is no speedometer on the starship Enterprise.

Stated this boldly, the idea is very counter to our experience. This is because what we generally observe as a velocity is not a velocity in space but is our velocity relative to the earth. Relative velocities are detectable. We note the amount of street that passes below our car or feel the flow of the air that moves over our face and infer a speed but we do not know how fast the earth is moving and thus do not know what our absolute velocity is. We do know that the earth moves around the sun and thus can determine our velocity relative to the sun. We know that the sun is moving in our galaxy and even that our galaxy is moving relative to other nearby galaxies and thus can know our velocity relative to the local cluster of galaxies. With the recent advances in astronomical detection, we are able to note our velocity relative to the place that we occupied in the early universe, our motion relative a background microwave radiation that is a detectable relic of the early universe, but again we cannot know whether that place had a velocity.

The inability to detect velocity is one of the most mysterious and counter intuitive concept that has ever been articulated. Consider a remote and empty part of the universe, no stars or galaxies nearby. Here there are no discernible forces and a released body moves in a straight line with a constant velocity. This is one of Newton's Laws and was his articulation of Galilean Invariance. Although when we start to work on General Relativity, we will have to revisit these issues, let us assume that this empty region is space. We envision this as that stable structure that Descartes and Newton needed as a background against which motion took place. In this day and age, it is generally easy to convince someone that this space obeys the Copernican Principle; it is not centered on some special place like the earth. It is also not difficult to convince someone that this idea should be extended to the

general Copernican Principle that, in an empty universe, there is no special place that could be called the center. This idea that there is a thing called space and that it is a stable structure and has no special places in it is better stated as the fact the the universe is homogeneous. Stated in a fashion that is similar to our statement of Galilean Invariance above, we can say that there is no experiment that can be performed in space that can distinguish one place from another. This is the definition of a homogeneous space. It should be obvious that if you cannot distinguish between places that you cannot have a center or a boundary. These are special places and this is contrary to the idea that all places are the same.

It might seem that these assumptions about the nature of space are so obvious that the universe must obey them. That is never the case in physics. You must test any hypothesis. On the other hand, you may want to say that this in not a hypothesis that is testable; we cannot be anywhere other than where we are. The best test of this idea is that we find that the laws of physics as we know them here on earth are found to be applicable everywhere that we apply them including distant space. Stars in remote galaxies operate in the same fashion as nearby stars. The laws of optics and electromagnetism are the same. We can also look at distributions of matter such as galaxies. Again, there is no indication that the universe is not homogeneous. A related concept is isotropy. This is the idea that space is the same in all directions. This hypothesis has been tested very precisely by the distribution of the microwave radiation that we observe.

Now consider two sets of physicists that are moving toward each other at some velocity, \vec{v} , and are studying the universe. If we now impose Galilean Invariance, each must have the same rules of physics and, thus, observe a universe that is homogeneous and isotropic. Yet, they are moving toward each other. It is not intuitive that space and time can be constructed consistently in this way but they are.

In other words, if we define the x direction as the line connecting the two sets of physicists, they will each measure events using space and time coordinates let's say (x, y, z, t) and (x', y', z', t') that satisfy the following relationships:

$$\begin{aligned}x' &= x - v_0 t \\y' &= y \\z' &= z \\t' &= t\end{aligned}\tag{9.1}$$

These are the Galilean transformations; the rules that indicate how to trans-

late one of the set of observer's observations to the other set of observer's observations. Each set of physicists, one set making measurements with (x, y, z) and t and the other with (x', y', z') and t' and each concluding that the universe is homogeneous and isotropic. Not only that but there is no experiment that they can perform that can yield a different result. If the physicists that differed in their measurements of events in space and time as given in equations 9.1 on page 216, found different rules for their experiments, we could tell them apart. If only one of them had Newton's Laws for the motion and the other did not, we would say that that one is at rest and the other one was moving. But since both sets of physicists have the same rules of physics and observe the same universe how can you tell which one is moving and which is at rest. **In summary: there is no experiment that can perform that can determine your velocity – all the laws of physics must be unaffected by a velocity choice.** All these relatively moving sets of observers have the same laws of physics as long as their velocity is unchanging.

By the way, it should be clear that although the different sets of observers must have the same results for any experiment, they will each describe the other's experiment differently. If one set of observers release a piece of chalk at rest relative to them, they will say that it remains at rest. Its coordinate is some (x_0, y_0, z_0) which is unchanging and its velocity is $\vec{v} = (0, 0, 0)$. The observers that are moving relative to this first set with a velocity $\vec{v} = (v_0, 0, 0)$, again choosing the x direction as the direction of relative motion, will say that the released chalk is moving uniformly in the direction of decreasing x but staying in the same place in the yz plane.

Said another way, Galilean invariance does not require that the different observers measure the same values for the things which they observe. Contrary, for the same experiment to produce the same result requires that the descriptors be different. For instance, the two observers give different descriptions of where an object is. An object that is at rest at the origin as measured by one observer will be seen as moving by the other observer. For one observer it will have non-zero kinetic energy and for the other it will have zero kinetic energy. Although places, velocities, kinetic energies are different, if the two observers do the same experiment, the same thing happens.

Consider two observers on the surface of the earth. This is not empty space and the local universe is not homogeneous and isotropic; you can tell up and down from sideways – things fall down because of gravity. But all the laws of physics must obey Galilean invariance including gravity. Have these observers move by each other at a uniform relative speed v in a horizontal

direction; both observers place a chalk on the end of their nose and release it. It falls and lands between their feet. The same experiment yields the same result. To each observer, the chalk falls along the line from his nose to his foot. Either observer when describing the others experiment sees the chalk with an initial velocity but cleverly arranged so that as it moves so does that other observers foot so that as the chalk drops past the observers foot, the foot is there also.

As stated before the principle of Galilean invariance is to state that there is no experiment that can be performed that can determine our velocity. This is true even in the presence of other fundamental forces such as gravity.

Acceleration, on the other hand, is detectable. Again consider our two observers on the surface of the earth with relative motion in the horizontal direction except in this case let there be an acceleration for one of the observers. If they drop chalk from their nose accelerated observer drops chalk, it does not land between his feet.

This is connected with the usual statement of Newton's force law. We say that force which is a push or a pull from some external agent acts to produce an acceleration according to $\vec{f} = m\vec{a}$. If there is no force the object does not change its velocity; it stays at rest to some set of equivalent observers. Not only does Galilean invariance effect the force free case, it is also operative when forces are present. In order to guarantee that all experiments have the same result, you have to make sure that \vec{f} does not change when you make the connections to the other relatively moving observers, i. e. make the Galilean transformation. It is only in this case that you can have Galilean invariance. For instance, if we are talking about the Hook's law spring, $\vec{f} = -k\vec{x}$, the force changes under the transformation of equation 9.1 on page 216. But this system does not represent a homogeneous space; There is a special place, $x = 0$. The spring is attached to a fixed point. In the real world the spring is attached to another mass, m_2 . In this case the law of force on the original mass is $m_1\vec{a}_1 = -k(\vec{x}_1 - \vec{x}_2)$. Now if you apply the transformation, $x'_i = x_i + v_0t$, for $i = 1, 2$, there is no change in \vec{a} and $\vec{x}_1 - \vec{x}_2$ and thus you have Galilean invariance. As you would expect the analysis of this situation using action is even more informative, see Appendix ?? on page ?? . See Section 7.4.2 on page 197. For simplicity of notation, consider a world with only one spatial dimension. the action for this case is

$$S(x_f, t_f, x_0, t_0; path) = \sum_{path, (x_0, t_0)}^{(x_f, t_f)} \left(m_1 \frac{v_1^2}{2} + m_2 \frac{v_2^2}{2} + k \frac{(x_2 - x_1)^2}{2} \right) \Delta t \quad (9.2)$$

Now applying our transformation, we get a change but it is only from the velocity terms and is thus the same case as for the free particle. In Section 7.4.2 on page 197, this case is analyzed in detail and it is seen that this family of transformations is an invariance.

In contrast to our inability to perform an experiment that can determine our velocity, it is easy to determine our acceleration. Consider a spring with a mass on the end. If we are accelerating, the stretch of the spring in equilibrium is different if we hold the spring along the acceleration direction or transverse. If we were in outer space at a distance from a massive body and held a plumb bob on the end of a string, the string would point to the massive body if we were not accelerating and would point to the side if we were accelerating. In the action analysis above, applying the transformation $x'_i = x_i + \frac{a}{2}t^2$ does not change the interaction term but does change the velocity parts and in a non-trivial way which means that there is no symmetry nor invariance. This is also consistent with the fact that even for free particles, accelerations are detectable.

9.3 Implications of and for Maxwell's Equations

All of the experiments involving electromagnetic phenomena up to the discoveries leading to quantum mechanics are described by the following local field theory and the associated force law:

$$\begin{aligned}
 \text{div}(\vec{\mathbf{E}}(\vec{\mathbf{r}}, t)) &= \frac{1}{\epsilon_0} \rho(\vec{\mathbf{r}}, t) \\
 \vec{\text{curl}}(\vec{\mathbf{E}}(\vec{\mathbf{r}}, t)) &= \frac{\Delta \vec{\mathbf{B}}(\vec{\mathbf{r}}, t)}{\Delta t} \\
 \text{div}(\vec{\mathbf{B}}(\vec{\mathbf{r}}, t)) &= 0 \\
 \vec{\text{curl}}(\vec{\mathbf{B}}(\vec{\mathbf{r}}, t)) &= \frac{1}{\mu_0} \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) - \frac{1}{\mu_0 \epsilon_0} \frac{\Delta \vec{\mathbf{E}}(\vec{\mathbf{r}}, t)}{\Delta t}
 \end{aligned} \tag{9.3}$$

and the force law:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{9.4}$$

where $\rho(\vec{\mathbf{r}}, t)$ is the charge per unit volume, $\vec{\mathbf{j}}(\vec{\mathbf{r}}, t)$ is the current density or charge per unit area per unit time, $\vec{\mathbf{E}}(\vec{\mathbf{r}}, t)$ is the electric field or force per unit charge, and $\vec{\mathbf{B}}(\vec{\mathbf{r}}, t)$ is the magnetic field or force per unit charge times speed. The force, $\vec{\mathbf{F}}$ is the force on a charged particle with charge q and velocity $\vec{\mathbf{v}}$.

This system of equations, describes the electric and magnetic force system as a local field theory. Local field theory in contrast to the action at

a distance theories of the 18th and 19th century has become the vehicle of choice for the description of fundamental phenomena. The basic ideas and the procedures associated with field theory approaches are introduced and reviewed in Chapter 5 on page 123.

Like any system of forces, this set of rules articulated by Maxwell's equations, Equation 9.3 on page 219, must obey Galilean invariance or we would be able to use electromagnetic phenomena to determine a velocity in space. For instance, if you do a careful analysis of the dimensional content of these equations you will find that ϵ_0 and μ_0 have dimensions and that the combination $\frac{1}{\sqrt{\mu_0\epsilon_0}}$ has the dimensions of a speed. In fact, this speed is the characteristic speed of travel for changes in the fields and this is the speed at which disturbances of the field, light, travels. If Maxwell's equations and the associated force law are correct in all frames, the two fundamental dimensional constants must be the same and, thus, the speed of changes in the electromagnetic field must be the same to all observers.

This situation with Maxwell's equations presented quite a quandary to 19th Century physicists. Since Maxwell's equations are not Galilean invariant in the sense that they are left unchanged by the transformation law of equations 9.1 on page 216, then velocity could be measured and light could be used to do it. In other words, there was some preferred state of uniform motion in which the Maxwell's equations were true as written and in this frame the measured speed of light was $\frac{1}{\sqrt{\mu_0\epsilon_0}}$. This is analogous to the case of the stretched string in which the rest frame of the string is the preferred state in which the dynamics takes on a simple form and the speed of the waves was set simply by the parameters of the dynamic. For the case of the Maxwell system, an observer moving at any velocity with respect to the frame with the simple dynamic would not measure the same speed for light and would also have to modify equations 9.3 on page 219 and 9.4 to account for the relative velocity and in that system the equations would contain the relative velocity as additional parameters. It was still a quandary though in that all other fundamental dynamical systems were Galilean invariant but not electromagnetic phenomena. In fact, we will see that it was Einstein's genius to go the other way and insist that there were no experiments that could determine a velocity but that the simple transformation law, equation 9.1 on page 216, had to be modified and that Maxwell's equations 9.3 on page 219 and the force law 9.4 on page 219 were correct.

An interesting feature of Maxwell system and the force law is that, from the way that it operates, the magnetic force only changes the direction of a particle. It cannot do any work. From the work energy theorem it follows

that it cannot change the kinetic energy of a particle that is subject to only magnetic forces. This is a paradox. We get all of our electrical power from dynamo that are operated by magnetism. Let's take a closer look at this problem:

Side issue on Gleeson's magnetic paddle

Consider an electron and a large massive magnet. Shoot the electron into the magnet at some speed v . It is deflected and comes out at the same speed that it went in at. This is very satisfying since the kinetic energy before and after is the same.

Now consider the situation in which the electron is initially at rest and the magnet is moving at the speed v toward the electron. Initially the electron has zero kinetic energy. After it encounters the magnet, the electron is moving away from the magnet at the speed $2v$. This is how any massive paddle works. If you hit a light particle with a massive elastic paddle the light object is moving forward with speed $2v$.

The striking thing about the magnetic paddle is that like any paddle, the light particle goes from having no kinetic energy originally to one that has kinetic energy. But magnetic fields do not do any work?

If we analyze the situation in the frame of the moving magnet we see immediately the resolution for this seeming paradox. In this frame there is not only a magnetic field but also an electric field. In fact, the electric field, \vec{E} , is perpendicular to \vec{B} and is directed along the sideways displacement that the charged particle experiences. In order to increase the kinetic energy of the charged particle to $2mv^2$ the electric field had to be $E = vB$. The important point is that you can use this paddle to convince yourself that under the Galilean transformation you not only change the coordinates but also have to change the fields \vec{E} and \vec{B} . If they are to recover the same laws of physics, what one inertial observers says is a magnetic field will be viewed as being both a magnetic and electric field to another inertial observer.

This example shows that our ideas about what changes are necessary to have invariance between moving observers will have to go beyond just coordinate changes: it must deal with rearrangements of the elements of coupled systems.

Return to Maxwell's Equations again

In a similar fashion to the case of the stretched string, see Appendix 5.3 on page 128, Maxwell's equations predict that in a source free region there are

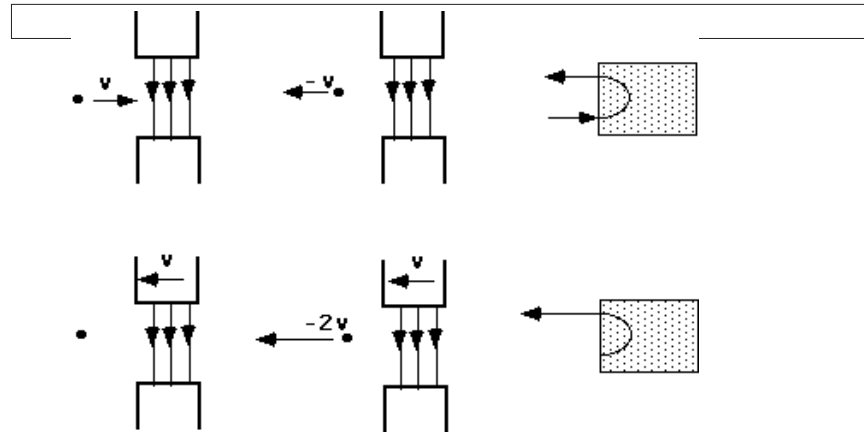


Figure 9.1: **The Magnetic Paddle:** In the upper part of the figure, a small charged particle represented by the dot is moving with speed v into a large magnet. It is deflected and comes out at the same speed v with which it entered. Now consider the same situation viewed from the frame in which the charged particle is initially at rest. Here the magnet is moving with speed v . After the magnet has passed over the original position of the charged particle, the particle is moving to the left at speed $2v$.

wavelike disturbances and the speed of these disturbances is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \quad (9.5)$$

If these equations have to be modified to account for relative motion to the special frame in which they are true, then there should be many ways to observe these effects and measure our velocity relative to the special frame.

Actually, this is not the case. The speed of light is very large compared to speeds of terrestrial relative motion. This means that it is generally difficult to detect the small corrections caused by the relative motion. Several clever experiments were undertaken to detect motion relative to the preferred frame. These are discussed in Section 9.4 on page 223. None of these were able to detect the effects that were expected and this series of experiments were called the search for the aether that was supposed to be the underlying machinery of the electromagnetic field. This frustrating effort reached its culmination in the definitive series of special experiments carried out by Michelson and Morley in the later part of the 19th, see [Whittaker 1953].

9.4 Pursuit of a special frame

The Maxwell's equations, Equations 9.3 on page 219, implied that there were field configurations of the electromagnetic fields that traveled. In all previous examples of these traveling solutions in continuous systems the speed of travel was measured relative to that of the medium. For example, the speed of the travelers in the stretched rope, $v = \pm\sqrt{\frac{T}{\rho}}$, was relative to the rest frame of the string. In fact for Maxwell to understand the dynamics of electromagnetic system, he had to invoke the underlying structure of vortices, Section 5.4 on page 140, and the velocity of the disturbances was relative to the rest frame of these vortices.

9.5 Michelson-Morley Experiment

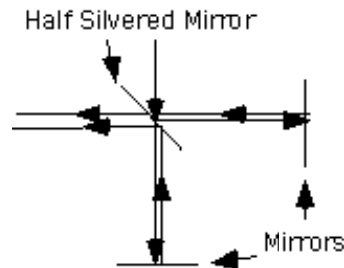


Figure 9.2: **Schematic Diagram of the Michelson-Morley Experiment:** Light enters the apparatus from above. It encounters a half silvered mirror so that one beam travels down to a reflecting mirror and returns to the half silvered mirror, reflects and leaves the apparatus to the left. The other beam from the half silvered mirror reflects to the right to a mirror and returns to the half silvered mirror to recombine with the first beam exiting to the left.

This experiment by the famous American physicist, Albert Michelson, was a search for the preferred frame for Maxwell's equations. It ultimately provided the experimental verification that there was no preferred frame and that the speed of light was the same for all relatively moving observers. It is important to point out that although this experiment is a direct verification of Einstein's postulates that are at the basis of the Special Theory of Relativity, Einstein was not aware of the experiment at the time he proposed

the theory. He based his argument on the nature of Maxwell's equations and their implications.

The fundamental idea is to try to detect an effect of the motion on the observed speed of light. It would be easy to just measure the speed of light for different states of motion and compare them. This is not possible because the speeds at which we can move an apparatus to measure the speed of light is generally negligible or well within the experimental error compared to the measurement of the speed of light itself. Michelson came up with a clever idea that would have allowed him to detect the small, actually large by most measures, speeds of celestial motion on the speed of light if they were there.

The basic idea is to compare the speed of light in two perpendicular directions at the same time, see Figure 9.2 on page 223. Since relative velocity is a vector or directed quantity, it will effect the speed of light differently in two different directions. This gets around the problem of making a direct comparison of the relative velocity to the speed of light.

To understand the experiment, let's look at a simple situation that is easier to understand.

The swimmer analogy

If a swimmer can swim at a speed v in still water and she wants to swim directly across a stream of width, D , that flows at a speed v_0 as shown in Figure 9.3 on page 224, she has to swim so that the resultant velocity, the vector sum of her velocity in the water and the velocity of the water, is directed across the stream.

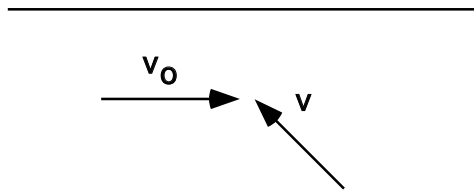


Figure 9.3: **Problem of a Swimmer in Flowing Stream:** A stream of width D is flowing with speed v_0 from left to right. A swimmer whose speed in still water is v wants to swim across and back, reaching the other bank at a point opposite the starting point.

The resultant velocity which is directed across the creek is thus $\sqrt{v^2 - v_0^2}$. If she wants to swim back again the total time is $2 \frac{D}{\sqrt{v^2 - v_0^2}}$.

If she wants to swim up the creek, with the current, a distance D and back the time is $\frac{D}{v+v_0} + \frac{D}{v-v_0}$. These two round trips cover the same distance

but the times are not the same. The difference of the round trip times is

$$\begin{aligned}\Delta t &= 2\frac{D}{v}\left(\frac{1}{1-\frac{v_0^2}{v^2}} - \frac{1}{\sqrt{1-\frac{v_0^2}{v^2}}}\right) \\ &\approx \frac{D}{v}\frac{v_0^2}{v^2},\end{aligned}\tag{9.6}$$

where I have used the relationship $(1+x)^n \approx 1+nx$ for $x \ll 1$. In the case of a swimmer, the ratio of the speeds, $\frac{v_0}{v}$ is a number a little less than one. Thus this difference in times is easily measured. It is also a fact that we could measure the speeds directly and just from the time swimming a distance D determine the drift of the stream.

Return to Michelson-Morley

For light if we say that there is no Galilean invariance and there is a special frame in which the speed of light is $\frac{1}{\sqrt{\mu_0\epsilon_0}}$. Then if we move relative to that frame, we should detect an effect on the speed of light to us. This is like the stream above. The speed of the swimmer is the speed of light in its preferred frame and the drift of the current is speed that we are moving relative to that preferred frame. The cleverness of the Michelson-Morley experiment is that it takes advantage of a special property of light to make it easy to measure the time differences for light traveling over different paths.

In the Michelson-Morley experiment, see Figure 9.2 on page 223, light enters the apparatus from above and is split into two beams by a half silvered mirror. One beam travels horizontally to a mirror and returns to the half silvered mirror and the other continues down to another mirror and returns to the half silvered mirror. The half silvered mirror allows the horizontal beam through and deflects the vertical beam so that the two beams can be combined and focused into an eyepiece. Thus if the apparatus is drifting in space at a velocity \vec{v}_0 relative to the frame in which the speed of light is c and also if the velocity is horizontal, we have the same circumstance as the swimmer. The net speed of the light in the two legs of the apparatus will be different and there will be a difference in time of transit through the apparatus. By using monochromatic light and the fact that the light is periodic with a very high frequency, Michelson and Morley can compare the arrival times with great precision.

From the Fresnel construction, Section ?? on page ??, and similar to the construction used to describe the Young's double slit experiment, Section ?? on page ??, the amplitude for the light at the eyepiece is the sum of the amplitudes from each leg of the apparatus. The phasors for each of these amplitudes rotate at the same frequency as the frequency of the light but will

have a different phase depending on which leg the light traveled over. In fact, the two phasors will have a difference in phase angle that is the difference in the travel time divided by the time associated with the characteristic frequency for that light or $\phi = \frac{\Delta t}{T} = \Delta t \times f = \Delta t \times \frac{c}{\lambda}$, where f is the frequency and λ is the wavelength of the light. Using “Things that Everyone Should Know”, see Section 1.4.2 on page 16, you can see that a very small Δt will produce measurable phase differences.



Figure 9.4: **Fringes of Michelson Morley Apparatus:** The pattern of bright and dark lines that are seen when viewing through the eyepiece of a Michelson interferometer. These patterns are called fringes and it was anticipated that the fringe pattern would shift as the Michelson–Morley apparatus was rotated.

Of course in the actual apparatus, it is impossible to make the two arms the same length to the necessary precision. This would require that they be equal to within a portion of the wavelength of the light used. But also, you should realize that over the width of the beam you cannot align the mirrors that precisely. What you actually get is a pattern of lines over the width of the beam. Bright lines where the phase difference is an even multiple of π and dark lines where the phase shifts are an odd multiple of π . The bright and dark pattern of lines is called fringes, see Figure 9.4 on page 226. If you now rotate the apparatus, the role of the velocity relative to the special frame will shift in the two arms of the apparatus and the slower leg will become the quicker leg and visa versa. The fringes will shift. Michelson and Morley could detect a fringe shift as small as $\frac{\pi}{4}$. Using the results of our swimmer analogy, if the apparatus has arms of length $D \approx 10$ meters, the light can be reflected several times in each leg, the ratio of the drift velocity to the speed of light that the apparatus can detect is the order of $\frac{v_0}{c} \approx \sqrt{\frac{\lambda \times \phi}{D}}$. Using $\phi = \frac{\pi}{4}$, this apparatus can detect a relative velocity that

is about 10^{-4} of the speed of light or $3 \times 10^4 \frac{\text{m}}{\text{sec}}$. This is still a very high velocity for the apparatus but fortunately it is about the speed of the earth in its orbit. Thus since the apparatus is in orbit on the earth, Michelson and Morley should have seen fringe shifts as they rotated their apparatus. Over long periods of time, there were effectively no fringe shifts. This experiment has been repeated many times and with even greater precision than that of Michelson and Morley and still no fringe shifts. This experiment is a direct test of the postulate that regardless of your state of motion, the speed of light is the same in all directions. This is essentially Einstein's postulate about the structure of space time that is the basis of the Special Theory of Relativity. In the following Chapter, we will develop the consequences of this postulate.

There were many other attempts to detect our motion relative to the special frame in which Maxwell's equations are correct. None of them were as definitive as the Michelson – Morley experiment and none of them have contradicted the postulates of the Special Theory of Relativity.

